Methodological skills

rMA linguistics, week 14

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This week

- ANOVA = Analysis of Variance
- Today: one-way ANOVA (M&M 12)
- Advanced stat course: two-way ANOVA (M&M 13)

Inference on means

Sample mean $\bar{x} \rightarrow$ population mean μ?

- One-sample t-test: is $\mu = \mu_0$?
- Two-sample t-test: is $\mu_1 = \mu_2$?
- More populations, more samples: analysis of variance: is $\mu_1 = \mu_2 = ... = \mu_I$? (ANOVA, "generalized t-test").

Comparing more populations

- Broca's vs. Vernicke's aphasia vs. controls.
- Verbs vs. nouns vs. adjectives vs. function words.
- French vs. English vs. German vs. Dutch.
- 2×3-design:

male/N	male/V	male/A
female/N	female/V	female/A

Two ways of looking at ANOVA

1. Compare more populations:

   to they have the same mean?

2. Nominal explanatory variable(s)
   explaining a quantitative dependent variable:

   How much of the variance in the dependent variable
   can be explained by the explanatory variable(s),

   and how much of this variance is within-group noise?

One-way ANOVA

1. Compare populations $1...I$:
   do they have the same mean?

2. One explanatory variable, with I levels:
   does it explain between-group differences?

- Populations can be classified in one way.
- Mean of responses in one-factor experiments.

Two-way ANOVA

1. Compare populations $1...I \times 1...J$:
   do they have the same mean?

2. Two explanatory variables, with I and J levels:
   do they explain between-group differences?
   Which one? Both? Interaction?

- Populations can be classified in two ways.
- Mean of responses in two-factor experiments.

Basics of ANOVA

- Populations $1, 2, ...I$.
- Single quantitative variable X on units/cases.
- Interested in pop means $\mu_1, \mu_2, ...\mu_I$ of var X.
- One sample for each population:
  - n_i: size of sample i $(1 \leq i \leq I)$.
  - x_{ij}: case j within sample i $(1 \leq j \leq n_i)$.

H_0 and H_a in ANOVA

- Null hypothesis: H_0: $\mu_1 = \mu_2 = ... = \mu_I$.
- Alternative hypothesis: not all of the μ_i are equal.
  That is, there exist some i and some j such that $\mu_i \neq \mu_j$.

Assumptions of ANOVA

- Fairly Normal distribution per subgroups, no outliers (use Normal quantile plot).
- Population standard deviations are equal:

  Instead of performing formal tests: if largest (sample) standard deviation $<$ $2\times$ smallest (sample) standard deviation, then you can safely use ANOVA.
- Independent observations.

  (Repeated measure ANOVA for test-retest situations!)

The ANOVA model

- Decompose:
  DATA (total) =
  = FIT (between group) + RESIDUAL (within group).

- F-distribution: reject H_0 if variation among groups is large relative to variation within group.

- (F-test for equality of spread/variance M&M 7.3: different from ANOVA, but also employs F-distribution. In general, F-tests are used for the ratio of two Normal distributions.)

Software output

	Sum of squares	df	Mean square	F	Sig.
Between groups	7.73	3	2.58	11.22	.001
Within groups	3.21	14	.23		
Total	10.94	17			

Reporting ANOVA results

... significant/not significant at $\alpha = 0.05$ level ($F(df1, df2) = ..., p = ...$).

- $df1$: degree of freedom "between groups" (fit, numerator).
- $df2$: degree of freedom "within groups" (residual, denominator).

If ANOVA null hypothesis rejected...

- At least one of the means is different from others. Which one?
- Prior (before data collection) vs. posthoc (after exploratory data analysis).
- (Prior) **contrast**: one-sample t-test with the null hypothesis that $\psi = \sum_{i=1}^{I} a_i \mu_i = 0$ for some a_i's depending on $a\ priori$ hypothesis ($\sum a_i = 0$).

If ANOVA null hypothesis rejected...

- **Multiple comparison**: pairwise comparison of samples i to j.
  - Large I: many comparisons performed.
  - Therefore, reduce α level.
    E.g., Bonferoni: guarantees that the probability of any false rejection no greater than original $\alpha = 5\%$.

ANOVA for the exam

When to use it:

- Recognize situations in which ANOVA needed.
- What is being tested (null hypothesis, alternative hypothesis).
- Criteria for its use.
- Interpret software output.

Mathematical details (ANOVA model) only for interested.

Next week:

- Final exam:

  Tuesday, May 29, starting at 11.00 in BH 015.

See you next week!